The Galton Machine and the Quest for a Robotics Simulator

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Abstract

The choice of a robotics simulator is a crucial decision for any research group, as it will affect their daily work for many years to come. This situation is getting more relevant with the additional use of simulators as modules embedded inside the robot's control architectures In this paper, we present an innovative methodology to evaluating robotics simulators, focusing on their fidelity to physical laws and computational requirements under rigorous tasks. To this end, we conduct quantitative comparisons using a Galton machine simulation experiment with the Webots, CoppeliaSim, and Gazebo Harmonic simulators. These comparisons allow researchers to make informed decisions in selecting the most suitable simulator for their needs.

Keywords: Robotics Simulators, Galton Machine, Control Architectures

1 Introduction

Simulation has long been integral to robotics research and development as a surrogate testing ground for algorithms and system behaviors. The value of simulation becomes even more pronounced as it provides a venue for high-accuracy experiments without the overhead costs and risks associated with real-world testing [1, 2]. In recent years, the utility of simulators has extended beyond mere experimental platforms. They are increasingly being embedded directly into robotic control architectures, serving as dynamic modules that inform real-time operation [3]

In this ever-evolving landscape, selecting a suitable robotics simulator is no longer a trivial task but a pivotal decision that can critically impact the effectiveness and efficiency of research operations for years to come [4–6]. The multitude of available simulators—each with its own set of features, physical fidelity, and computational demands—only compounds this challenge. Despite the increasing importance of simulator choice, the research community lacks a standardized framework for evaluating and selecting simulators based on their adaptability as embedded modules and their performance metrics [6].

Addressing the absence of a unified approach to evaluating robotic simulators, this paper aims to bridge the gap by outlining one of the essential criteria that simulators must meet to integrate seamlessly into control architectures. These criteria encompasses not only the fidelity with which physical laws are mimicked, but also the computational efficiency as a crucial factor for real time applications. To enable a rigorous and unbiased comparison between various simulators, we introduce a pioneering methodology that quantitatively evaluates them on these two dimensions.

As a cornerstone of our evaluation framework, we employ the Galton machine as a case study and benchmarking tool [7]. Though ostensibly simple, the Galton machine is a robust litmus test for simulators, challenging them on multiple fronts. On the computational side, its stochastic nature demands high numerical accuracy and low-latency calculations to produce believable simulations. Physically, simulating the interactions among numerous particles in the Galton machine requires rigorous and precise algorithms capable of capturing complex dynamics. Thus, the machine offers a multifaceted but compact problem that provides insights into a simulator's capabilities and limitations, thereby enabling an informed choice for researchers and developers alike.

The remainder of this paper is organized as follows: In Section 2, we explore the role of simulators in robotics development, from their evolution and multifaceted applications to their essential role as embedded modules. Section 3 introduces the Galton board and our approach's main advantages and drawbacks in evaluating robotics simulators. Section 4 provides a literature review, surveying the existing approaches to robotic simulation and identifying gaps that justify the necessity of this study. Section 5 introduces our novel methodology for evaluating these simulators, followed by Section 6, which describes the experimental setup, including the simulators tested, and the subsequent results. Finally, Section 7 offers conclusions and recommendations for future research

2 Simulators as embedded modules in cognitive architectures

Simulators are the cornerstone of efficient robotics development. Simulation technology has evolved to its current state based on parallel advances in computer hardware, graphics cards, mathematical software, rendering, gaming and other related disciplines. The original use of simulators as a digital substitute for physical reality to test designs before building them and to search for better designs has been extended in two ways: as a training ground for machine learning algorithms, and as a critical component of cognitive architectures for robotics. A wide variety of commercial and free simulators can be selected for these purposes. In all cases, two features are decisive in ordering the possible choices: the fidelity to the physical laws they implement, and the computational efficiency. The evaluation of these aspects is the subject of the next sections.

The main reason for including a simulator in a robot's brain is that it is now widely accepted that intelligent machines are predictive machines [8][9][10]. In humans, many regions of the brain - visual cortex, cerebellum, basal ganglia - can learn to predict -visual input, motor actions, and rewards, respectively. This means that the robot's brain should maintain a dynamically changing internal representation of the world by learning models of it as it acts in the real world. Once the robot has models, it will use them to predict the consequences of its actions and the actions of other agents [11][12]. The inevitable prediction failures will serve as triggers to improve the models and learn new ones. This is where simulators could play a crucial role in building these brains, as a piece of software that can efficiently run the internal models while remaining consistent with the underlying laws of Physics. This simulator use requires it to be partly synchronized with the robot's physical activity and partly ahead of it, anticipating what will come. A direct consequence of this functionality is a sense of normality, i.e., the ability to detect perceptual anomalies and trigger learning procedures. Other functionalities embedded simulators provide, such as *gravity filter*, the reduction of noise in the perception of objects' poses, the solution of occlusion problems in tracking, or the interpretation of others' gestures and intentions by replaying them in the robot's own model [13].

In order to be embedded in a control architecture, a robotic simulator must meet a number of specific criteria, most of which have not been a priority for its designers. The most important requirement is that it must be externally controllable very efficiently, providing a well-designed API via standard middleware. This API should provide creation, editing, and removal operations on all scene graph elements. It should also provide control over the simulation process, allowing it to be paused, reset, or the ratio of real to simulated time to be changed. It should also have a small memory footprint and CPU usage in order to fit into the robot's on-board computers. The tests carried out on the simulators used in this work include designing and coding a bridge component that can be used to interact with them as a replacement for reality and as an embedded module. The results of these implementations are discussed in the following sections.

3 Background: The Galton Board

The *Galton Board*, often referred to as a *bean machine*, is a device invented by Sir Francis Galton in the 19th century to demonstrate the central limit theorem and the normal distribution in statistics [7]. The apparatus consists of a vertical board with a grid of pegs, where balls (or beans) are dropped from the top and, as they bounce off the pegs in a random fashion, accumulate in slots or bins at the bottom (see Fig. 1).

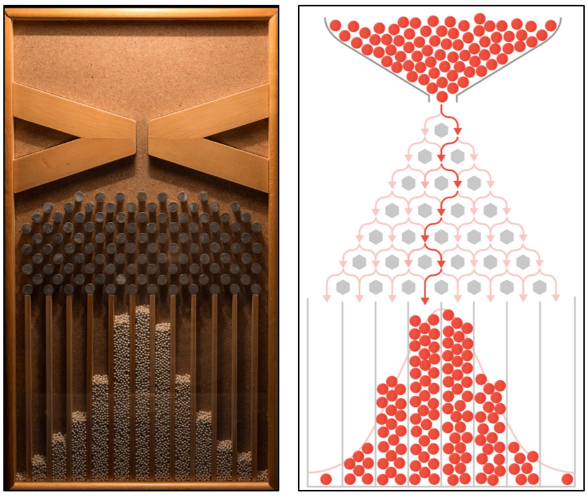

Fig. 1: Illustration of the Galton Boards and the Stochastic Pathways: A faithful reproduction of the original Galton machine is presented on the left. On the right, a schematic of a 8-bin Galton machine is displayed for visualization the stochastic paths.

Each ball follows a stochastic path, determined by the sequence of pegs it encounters. As many balls are dropped, the accumulated pattern in the bins tends to resemble a bell-shaped curve or the normal distribution. This visualizes the law of large numbers and provides an intuitive understanding of statistical concepts.

In the context of robotic simulations, the Galton Board emerges as a compelling benchmark. It offers a distinctive array of challenges, especially in terms of mimicking intricate physical interactions and inherent stochastic behaviors. This complex test bed serves as a rigorous evaluative measure for a simulator's physical fidelity and computational efficiency, themes that are elaborated in the ensuing sections of this manuscript.

With our proposal, **we aim to establish a metric for the behavior of the robotic simulator that quantifies both its approximation to physical reality and it's performance during the experiment's execution**. Employing this method offers the advantage of allowing an assessment of the simulator's performance and accuracy through an experiment that is straightforward to implement, grounded in statistical phenomena observable in reality. These phenomena arise from their inherent complexity and the numerous subtle factors influencing them. Thus, it's possible to evaluate the capability of the simulator's physical model through this method. Additionally, this metric does not require a deep understanding of the intrinsic workings of the physical engine, simplifying the experimental implementation itself.

Some studies have established that the radius of the cylinders of the Galton machine, as well as the radius of the spheres, are not significant factors in analyzing the results of the experiment [14], provided that the following initial conditions are met:

- The balls should not come into contact with the vertical boundaries of the Galton board.
- Each ball must experience multiple collisions with the nails before settling into one of the rectangular compartments.

However, the collision elasticity between the balls and the cylinders is very significant [15]. This reinforces the idea that when using the Galton machine as a benchmarking measure the specificity of the machine construction itself is of little value as long as the initial conditions are satisfied. The properties governing collisions, friction and coefficient of restitution are predefined by the simulator and these are what are tested with our proposal.

This experiment also presents its own set of limitations and challenges. One such challenge relates to the absence of direct knowledge regarding the behavior of the physical engine within the simulator. This lack of insight complicates the detection of flaws either in the experiment's execution or in the user-created simulation model. As a result, it becomes imperative to exercise critical scrutiny during the specific implementation of the experiment, particularly when observed outcomes deviate from expectations or when the simulator exhibits anomalous behavior. By acknowledging these limitations and incorporating them into our evaluative framework, we aim to contribute to the ongoing efforts in the scientific community to develop more accurate, efficient, and transparent robotic simulators. Thus, **this work thereby aspires to set a new standard in evaluating and comparing robotic simulation environments**, enriching the toolkit available to researchers in this ever-evolving field.

4 Related Work

The realm of robotic simulation has undergone significant transformations over the years, offering a broad spectrum of solutions tailored to diverse application areas. This section aims to provide an in-depth review of existing robotic simulation approaches and shed light on the areas where our work adds value.

A variety of robotic simulators such as Webots [16], CoppeliaSim [17], and Gazebo [18] have gained widespread recognition in both academia and industry. These simulators offer rich features, including high-fidelity physics engines, sensor emulation, and real-time simulation capabilities. However, most of these platforms primarily focus on the functionality and visual realism [4] they can offer, often sidelining computational efficiency and control architecture integration.

Several studies have attempted to define metrics for evaluating the performance of robotic simulators [19]. However, these metrics are often confined to specific use cases such as whole-body motion in humanoid robots [20] or grasping performance [19, 21]. There is a conspicuous lack of a generalized metric that can be applied to various robotic scenarios while considering both physical accuracy and computational efficiency. Besides, as robotic simulators find applications beyond mere offline testing environments, their ability to integrate seamlessly into the robotic architecture has become increasingly important [22]. Yet, a standardized approach to this end is largely missing, limiting the simulators' potential as embedded modules in complex robotic systems.

Some research initiatives have proposed benchmarks using real-world robotics tasks [19], but these often require elaborate setup and are not necessarily generalizable. The use of common scientific or mathematical problems as benchmarks, like the Galton machine, is underexplored in the existing literature.

Despite the strides made in the field, several gaps still remain:

1. **Lack of a Comprehensive Metric**: There is a lack of a holistic metric that accounts for both the physical fidelity and the computational efficiency of robotic simulators.
2. **Integration Challenges**: Limited work has been done on the criteria required for simulators to be easily integrated as embedded modules in robotic control architectures.
3. **Generalizable Benchmarks**: Existing benchmarks are often too specialized and do not lend themselves to general applications across different robotic simulators.

Our study aims to bridge these gaps by introducing a comprehensive set of requirements for robotic simulators, especially focusing on their role as embedded modules in control architectures. We also propose a novel, generalizable benchmark—the Galton machine—that serves as a rigorous yet accessible evaluative tool for assessing both physical fidelity and computational performance. In doing so, we seek to set a new standard for evaluating robotic simulation platforms.

5 Design of the experiments

In our endeavor to benchmark the fidelity of various robotic simulators, we have identified the necessity for a robust and statistical measure to evaluate the outcomes. To this end, we have chosen the chi-squared (χ^2) test. This statistical test is instrumental in comparing the observed and expected outcomes, quantifying the discrepancies. The chi-squared test will be formulated as follows:

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} \tag{1}$$

where O_i and E_i represent the observed and expected frequencies, respectively, for each end box of the Galton board in the simulation. We plan to conduct the experiment across three prevalent robotic simulators:

1. **Gazebo Harmonic**: The 8th major release of Gazebo simulator, Gazebo Harmonic offers a robust physics engine and supports various sensors and actuators. It is highly modular and offers a vast selection of pre-made models. It is often used in research applications requiring complex and realistic simulations, especially outdoor environments.
2. **Webots**: Known for its user-friendly interface and robustness, Webots is an open-source robotic simulator that offers a wide range of features including prototyping, development, and realistic rendering. It has the added benefit of supporting numerous programming languages and is employed in both educational and industrial research settings.

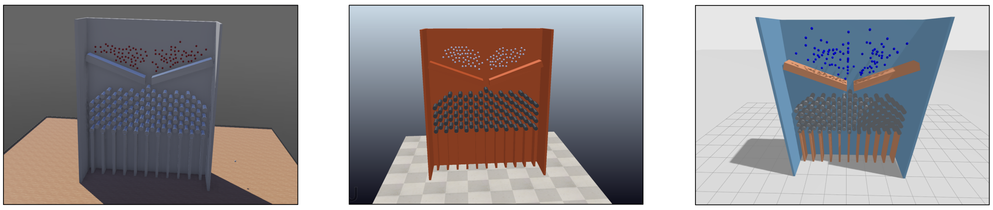
(a) Webots (b) CoppeliaSim (c) Gazebo Harmonic

Fig. 2: Initial scene for each simulator

3. **Coppelia Sim**: Formerly known as V-REP, Coppelia Sim is a versatile and scalable robotic simulator. It is unique in its ability to simulate robots and mechatronic and biomechanical systems. It also boasts a hybrid control architecture, allowing for both low-level and high-level control schemes.

It's imperative to note that each robotic simulator has its inherent assumptions, approximations, and algorithms to solve the dynamics. Therefore, the choice of the robotic simulator can substantially influence the simulation's fidelity and computational efficiency. By using the χ^2 test, we aim to derive meaningful comparisons and insights about their capabilities and potential limitations.

Our methodology is designed to provide insights into the simulation environments' robustness, fidelity, and reliability. We believe this will serve as a comprehensive guide for researchers and developers in selecting the most suitable simulator for their specific applications.

For this purpose, a scenario will be developed for all simulators, consisting of a 3D model of a Galton board with 11 rows of pins and 12 end boxes (see Fig. 2). At the same time, it should be noted that the balls used will be primitive spheres of each of the corresponding simulators. All simulations will be repeated a total of 10 times, making minor adjustments to the positions of the balls and the Galton machine to ensure distinct initial conditions in each iteration of the experiment. Furthermore, attention will be paid to any balls lost during the simulation due to collision errors.

Considering the chi-squared distribution, the following considerations were made:

- **Degrees of Freedom:** The degrees of freedom were set at **11**. This is calculated by subtracting 1 from the number of end boxes. The rationale behind this is that if we know the number of balls in 11 of the boxes, the 12th box is determined; hence, the degrees of freedom are 11.
- **Significance Level:** A significance level of **5%** was used. This is a commonly chosen value in statistical tests and indicates that there's a 5% risk of concluding that a difference exists when there is no actual difference.
- **Critical Value:** The critical value was determined to be **19.68**. This value is chosen based on the chi-squared distribution table corresponding to the desired significance level and degrees of freedom.

For the evaluation of the Real Time Factor (RTF) different experimental tests have been carried out in a 3D scenario consisting of a Galton board, varying the initial number of balls released. Experiments were conducted on two computers with different hardware capabilities: one with high performance and the other with more modest graphical capabilities. The detailed characteristics of each computer are presented in Table 1.

Specification	**Computer 1**	**Computer 2**
Processor	Intel i9-9900K CPU @ 3.60GHz	Intel i9-12900F CPU @ 5GHz
RAM Memory	G-Skill F4-3600C17-16GTZR x4	Kingston KF552C40 (16+32Gb)
GPU	NVIDIA GeForce GTX 1660 Ti	NVIDIA GeForce RTX 3090
Storage	SSD 1TB NVMe M.2.	Samsung SSD 980 PRO 1Tb
Operating System	Ubuntu 22.04	Ubuntu 22.04

Table 1: Computer Specifications

For the experimental setup, a three-dimensional model of the Galton board was designed using Blender modeling software. Subsequently, the model was exported using the .dae format (see [23]) and following the specific guidelines of each simulator, the mesh was imported into the respective simulation platforms.

6 Experimental Results

In this section, we present the results of our experiments conducted across the three simulators: Webots, CoppeliaSim, and Gazebo Harmonic. These results are the foundation for our comparative analysis, focusing on the chi-squared (χ^2), the percentage of lost balls (balls that "disappear" from the scene due to physics calculation errors by the simulators) in each simulation environment, and the RTF calculated in each simulator and computer for experiments with different numbers of balls. The experimental data is encapsulated in Table 2, Table 4 and Figure 3.

Table 2: Results for each simulator

Simulator	Chi-squared Value	Lost Balls (%)
Webots	16.64	0.4
CoppeliaSim	61.64	0
Gazebo Harmonic	54.84	54.3

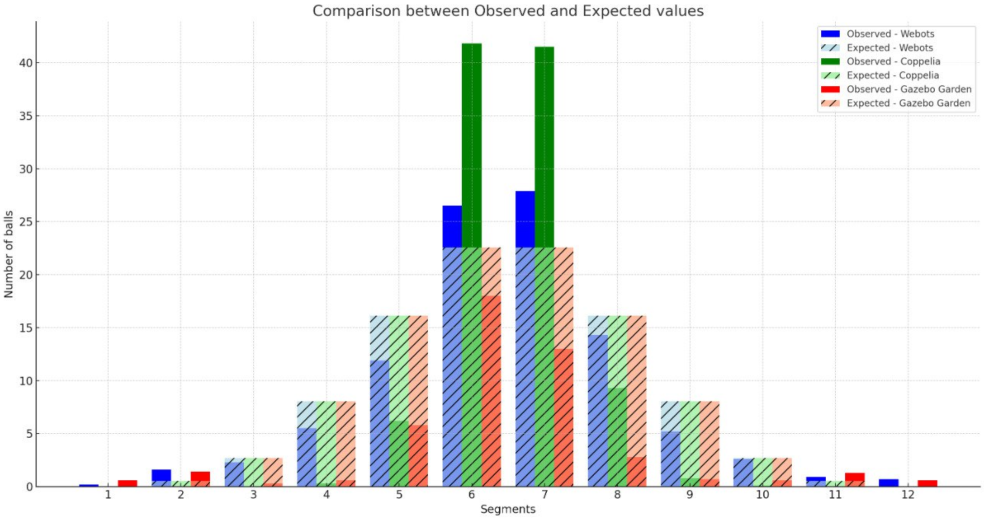

Fig. 3: Chi-squared distribution for each simulator

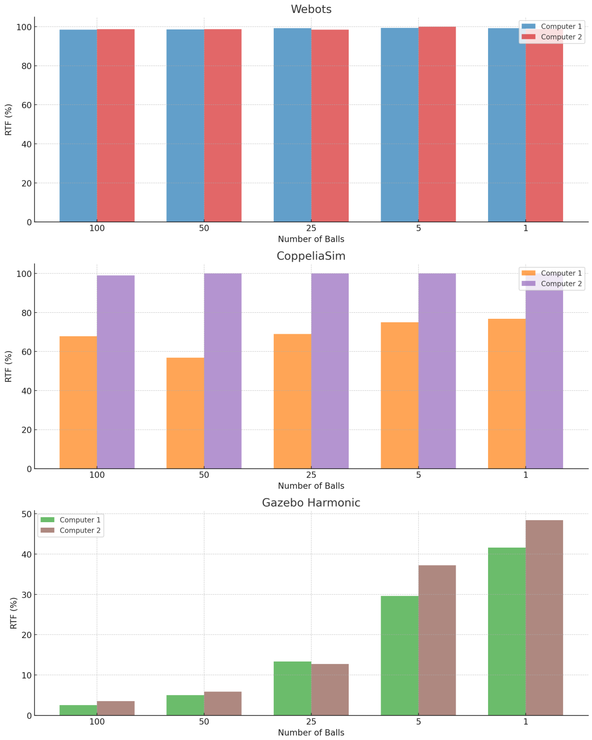

Fig. 4: Real Time Factor for each simulator

6.1 Discussion

Chi-Squared Value:

- **Webots**: The chi-squared value of 16.64 is below the critical value of 19.68, suggesting that the simulator provides a relatively accurate representation of real-world physics.
- **CoppeliaSim**: The value of 61.64 far exceeds the critical value, indicating a significant deviation from expected outcomes. [1]
- **Gazebo Harmonic**: With a value of 54.84, this simulator also deviates substantially from the expected results.

Lost Balls:

- **Webots**: Minimal ball loss (0.4%) shows high fidelity in collision detection.
- **CoppeliaSim**: No balls were lost, signifying precise collision detection but possibly at the expense of simulation fidelity, as indicated by its high chi-squared value.
- **Gazebo Harmonic**: The large percentage of lost balls (54.3%) suggests severe limitations in collision detection and raises questions about the simulator's reliability.

Real Time Factor:

- **Webots**: This simulator showed consistently high performance on both computers, with an RTF close to 100% for all ball counts. These results suggest that Webots can effectively handle the Galton machine simulation, regardless of the number of balls, without suffering significant performance degradation.
- **CoppeliaSim**: On computer 1, the CoppeliaSim RTF showed moderate variability, ranging from 57% to 77%, depending on the number of balls in the simulation. On the other hand, computer 2 maintained a constant RTF of 100% in all conditions. These results reflect the significant influence that hardware specifications and capabilities can have on the performance of this simulator.
- **Gazebo Harmonic**: Of the three simulators, Gazebo Harmonic showed the greatest variability in RTF, especially as the number of balls increases. As the number of balls decreases, the RTF increases, peaking at about 48% with a single ball on computer 2. These results suggest that Gazebo Harmonic may not be the most efficient simulator for this particular experiment, especially when there is a large number of balls. [2]

7 Conclusions and future works

Using a Galton board scenario, this study aimed to rigorously evaluate the fidelity of three popular robotic simulators—Webots, CoppeliaSim, and Gazebo Harmonic. Leveraging the chi-squared (χ^2) statistical test, we could assess the fidelity and capture collision detection and reliability robustness in these simulation environments.

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[1] The version of Coppelia used for the experiment was 4.3.0 EDU. The same experiment was run from version 4.5.1 EDU and an anomalous behavior was found that did not allow the experiment to be performed.

[2] To minimize the risk of using an unstable version of its physics plugins due to the recent release of Harmonic at the time of writing this paper, the experiment has been repeated on the previous version of Gazebo, Garden. There were no significant changes in the results.

The key findings of our experiments can be summarized as follows:

- Webots demonstrated the highest fidelity in simulating the Galton board, evidenced by a chi-squared value below the critical level and minimal ball loss. In terms of performance, Webots consistently maintained an RTF close to 100% across both computers, showcasing its efficiency and robustness in handling the Galton board simulation.
- CoppeliaSim, although experiencing no ball loss, had a significantly higher chi-squared value, suggesting fidelity issues in this simulation scenario. Performance-wise, CoppeliaSim's RTF was highly dependent on the computer used; while it achieved a consistent 100% RTF on the superior computer, it showed variability on the lesser hardware, indicating the importance of strong computational resources for this simulator.
- Gazebo Harmonic was the least reliable, with substantial ball loss and a high chi-squared value, indicating both poor fidelity and collision detection reliability in the context of the Galton board. Additionally, its RTF values exhibited the most variability among the simulators, especially when the number of balls increased, underscoring potential efficiency issues.

These results provide key insights into the strengths and weaknesses of each simulator in terms of both the fidelity of the physical simulation and the reliability of collision detection algorithms, as well as their performance efficiency. Future work could further explore these aspects to understand the underpinnings of the observed discrepancies.

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